HEAT SINK BASE SPREADING RESISTANCE OPTIMIZATION FOR ACHIEVING BETTER THERMAL PERFORMANCE

INTRODUCTION

Heat sinks are routinely used in electronics cooling applications to keep critical components below a recommended maximum junction temperature. The total resistance to heat transfer from junction to air, R_{ja}, can be expressed as a sum of the following resistance values as shown in Equation 1 and displayed in Figure 1.

$$R_{ja} = R_{jc} + R_{TIM} + R_S + R_f + \frac{1}{2\dot{m}c_p} \qquad (1)$$

$$R_f = R_{cond} + R_{conv} \qquad (2)$$

Where, R_{jc} is the internal thermal resistance from junction to the case of the component. R_{TIM} is the thermal resistance of the thermal interface material. R_f is the total thermal resistance through the fins. The final term in Equation 1 represents the resistance of the fluid, e.g. air, going through the heat sink where m is the mass flow rate and C_p is the heat capacity of the fluid. As Equation 2 shows, R_{cond} and R_{conv} are the conduction and convection resistance respectively through the heat sink fins respectively.

R_s stands for the spreading resistance that is non-zero when the heat sink base is larger than the component. The next few sections show the full analytical solution for calculating spreading resistance, followed by an approximate simplified solution and the amount of error from the full solution and finally the use of these solutions to model and optimize a heat sink.

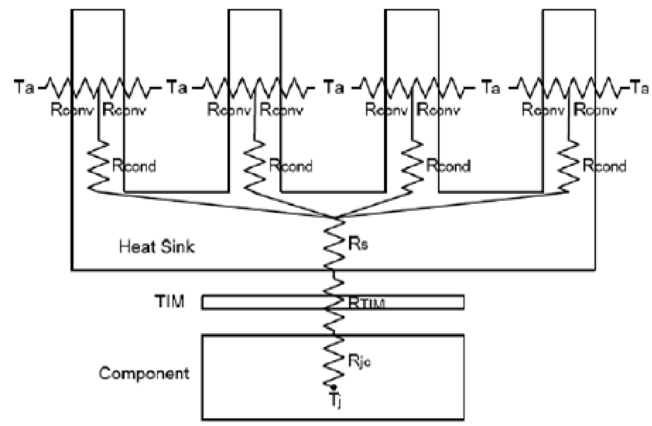

Figure 1. Resistance Network of a Typical Heat Sink in Electronics cooling [1]

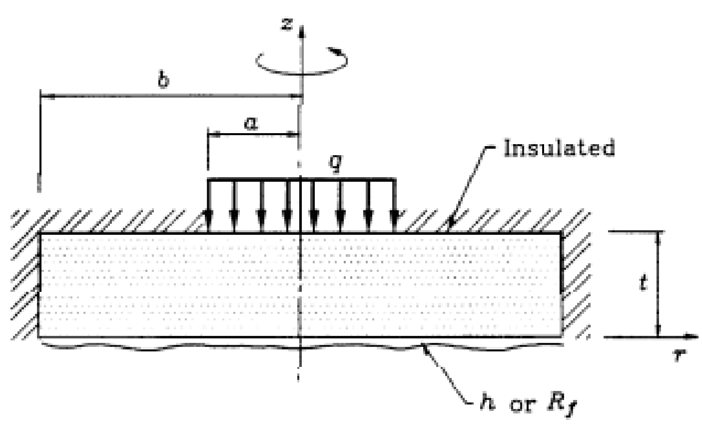

Figure 2. Heat Source on a Heat Sink Base [2]

ANALYTICAL SOLUTION OF SPREADING RESISTANCE

Lee et al. [2] derived an analytical solution for the spreading resistance. Figure 2 shows a cross-section of a circular heat source with radius a on the base with radius b and thickness t. The heat, q, originates from the source, spreads out over the base and dissipates into the fluid on the other side with heat transfer coefficient, h. For heat transfer through finned heat sinks, the effective heat transfer coefficient is related to thermal resistance of the fins, R_f as shown in Equation 3. For square heat source and plates, the values of a and b can be approximated by finding an effective radius as shown in equations 4 and 5.

$$h = \frac{1}{R_f A_{fins}} \tag{3}$$

$$a = \sqrt{\frac{A_{heater}}{\pi}} \tag{4}$$

$$b = \sqrt{\frac{A_{base}}{\pi}} \tag{5}$$

Where,
h = heat transfer coefficient [W/m^{2}K]
a = effective radius of the heater [m]
A_{heater} = area of the heater [m^2]
b = effective radius of the heat sink base [m]
A_{base} = total area of the heat sink base [m^2]

The derivation of the analytical solution starts with the Laplace equation for conduction heat transfer and applying the boundary conditions. Equation 6 shows the final analytical solution for spreading resistance. The values for the eigenvalue can be computed by using the Bessel function of the first kind at the outer edge of the plate, r=b as shown in Equation 7.

$$R_S = \frac{t}{kb^2\pi} + \frac{2}{ka}\left[\sum_{n=1}^{\infty} \frac{J_1^2\left(\frac{\lambda_n a}{b}\right)}{\lambda_n^3 J_0^2(\lambda_n)}\right]\left[\frac{\tanh\left(\frac{\lambda_n t}{b}\right) + \frac{k\lambda_n}{hb}}{1 + \frac{k\lambda_n}{hb}\tanh\left(\frac{\lambda_n t}{b}\right)}\right] \tag{6}$$

$$J_1(\lambda_n) = 0 \text{ @ } r = b \tag{7}$$

Where,
k = Thermal Conductivity of the plate or heat sink [W/mK]
J_1 = Bessel function of the first kind
λ_n= Eigenvalue that can be computed using Equation (3) at r = b
t = thickness of the heat sink base [m]

Lee et al. [2] also offered an approximation as shown in Equation 8 along with the approximation for the eigenvalues as shown in Equation 9. This approximation eliminates the need for calculating complex formulas that involve the Bessel functions and can be computed by a simple calculator.

$$R_S \approx \frac{1}{ka\pi}\left(\frac{at}{b^2} + \left(1 - \frac{a}{b}\right)\left[\frac{\tanh\left(\frac{\lambda_c t}{b}\right) + \frac{k\lambda_c}{hb}}{1 + \frac{k\lambda_c}{hb}\tanh\left(\frac{\lambda_c t}{b}\right)}\right]\right) \tag{8}$$

$$\lambda_c = \pi + \frac{b}{\sqrt{\pi}a} \tag{9}$$

APPROXIMATION VS FULL SOLUTION

Simons [3] compared the full solution (Equations 6 and 7) with the approximations shown in (Equations 8 and 9). The problem contained a 10 mm square heat source on a 2.5 mm thick plate with a conductivity of 25 W/mK, 20 mm width and varying length, L as shown in Figure 3. Figure 4 shows that the % error increases with length but stays relatively low. Less than 10% error is expected for lengths up to 50 mm; five times the length of the heater. This is acceptable for most Engineering problems since analytical solutions are first-cut approximations that should later be verified through empirical testing and/or CFD simulations. However, the full analytical solution should be used if the heater-to-heat sink base area difference gets much larger or if a more accurate solution is desired.

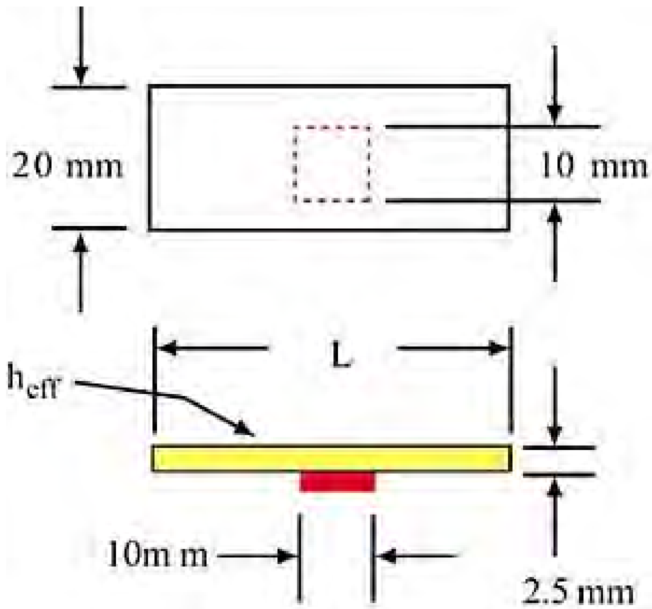

Figure 3. Example Problem For Comparing Analytical and Approximate Solutions for Spreading Resistance [3]

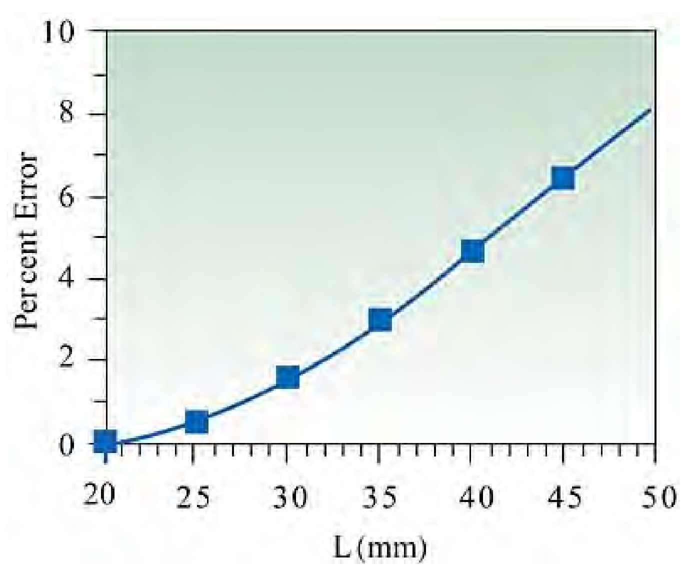

Figure 4. Percent Error Between the Analytical and the Approximate Solution of Spreading Resistance for the Example Shown in Figure 3 [3]

OPTIMIZING HEAT SINK PERFORMANCE

The goal of any electronic cooling solution is to lower the component junction temperature, T_j. For a given R_{jc} and R_{TIM}, the objective is to maximize heat sink performance by reducing the spreading resistance, R_s, and the fin resistance R_f.

The spreading resistance can be reduced by increasing base thickness. However, most electronics applications are limited by total heat sink height and thus any increase in base thickness leads to shorter fins which reduce the total area of the fins A_{fins}. For a fixed heat transfer coefficient (the heat transfer coefficient is a function of fin design and air velocity and we can assume it is fixed for this exercise) a reduction in the fin area increases R_f as shown in Equation 2. Equation 10 shows this combined heat sink resistance, R_{hs}, as a function of the spreading and fin resistance.

$$R_{hs} = R_s + \frac{1}{hA_{fins}} + \frac{1}{2\dot{m}c_p} \qquad (10)$$

Thus, for a given fin design, the thermal engineer must choose the appropriate heat sink base thickness to optimize heat sink performance. To illustrate this point, let's take an example of an application with the parameters as shown in Table 1.

Heat Source Size (mm)	10
Heat Sink Base Length and Width (mm)	50
Base Thickness (mm)	1-10
Material Conductivity (W/mK)	200,400
Total Heat Sink Height (mm)	20
Number of Fins (per 25mm width)	7
Heat Transfer Coefficient (W/m^{2}K)	50

Table 1. Example Heat Sink Application

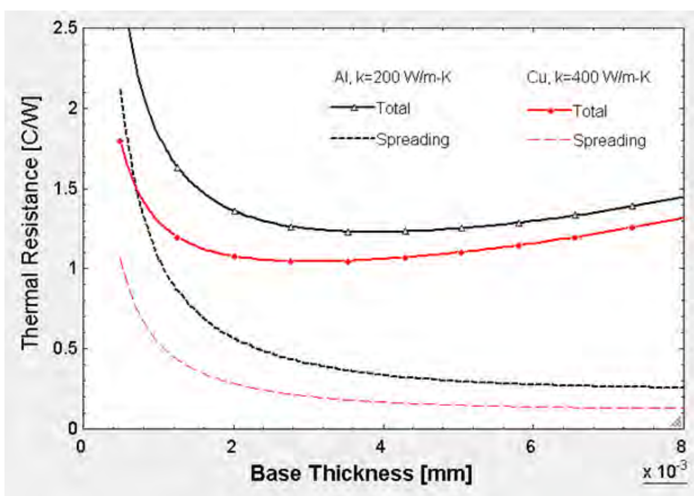

Figure 5. Total and Spreading Resistance of the Example Shown in Table 1 for a 50 mm Heat Sink

Figure 5 shows a graph of the total thermal resistance of the heat sink, R_{hs} and spreading resistance, R_s as a function of base thickness for copper and aluminum material. (Note that the final term from Equations 1 and 10 is ignored because it is constant and does not contribute to the understanding of spreading resistance). The graph shows that spreading resistance improves monotonically with increased base thickness. However, the total thermal resistance has an optimal point between 2-4 mm base thicknesses. For base thicknesses less than 2 mm, there is a sharp increase in spreading resistance which leads to a higher overall thermal resistance. On the other hand, increasing the base thickness above 4 or 5 mm gives diminishing marginal returns; the improvement in spreading resistance is minimal compared to the increase in thermal resistance due to the reduced fin area. Additionally, the graph also shows that higher conductivity materials such as copper, improves thermal performance across the entire domain.

CONCLUSION

The heat spreading resistance is an important factor when designing a heat sink for cooling electronics components. The full analytical solution for calculating the spreading resistance, shown in Equations 6 and 7, can be substituted with the approximations shown in Equations 8 and 9 with minimal error. The error increases with increased difference between the heat sink base and heater size and the complete analytical model should be used if needed. The analytical model can be used to choose the right heat sink base thickness that optimizes heat sink performance as shown in Figure 5. Techniques such as higher conductivity materials, embedded heat pipes, vapor chambers etc. are available if the spreading resistance is major obstacle in the cooling. Thermal engineers must balance the increased weight and cost of such techniques against the benefits for each application.

REFERENCES

1. “Spreading Resistance of Single and Multiple Heat Sources”. Qpedia. September 2010

2. Seri Lee et al. “Constriction/Spreading Resistance Model for Electronics Packaging” 1995.

3. Simons, Robert http://www.electronics-cooling.com/2004/05/simple-formulas-for-estimating-thermal-spreading-resistance/