Discharge Coefficient

in Flow Calculation and Measurement

INTRODUCTION

Measuring the volumetric flow rate in tubes, pipes and other conduits is an important task in fluid mechanics. One technique is to use a specially designed nozzle or orifice plate (a plate with a hole in its center) to restrict fluid flow. By measuring the pressure drop across this restraint, and using basic fluid mechanics principles, one can determine the volumetric flow rate or velocity of a fluid passing through a tube or pipe. An important element in this process is the discharge coefficient, which is defined as: "In a nozzle or other constriction, the ratio of the mass flow rate at the discharge end of the nozzle to that of an ideal nozzle which expands an identical working fluid from the same initial conditions to the same exit pressure" [1]. Essentially, the discharge coefficient is a means of accounting for the non-ideal losses associated with friction and other effects that are inherent to pressure differential flow measurement devices.

Theory and Discussion

To better understand the role of the discharge coefficient, it is helpful to look at a typical flow meter's construction, as shown in Figure 1.

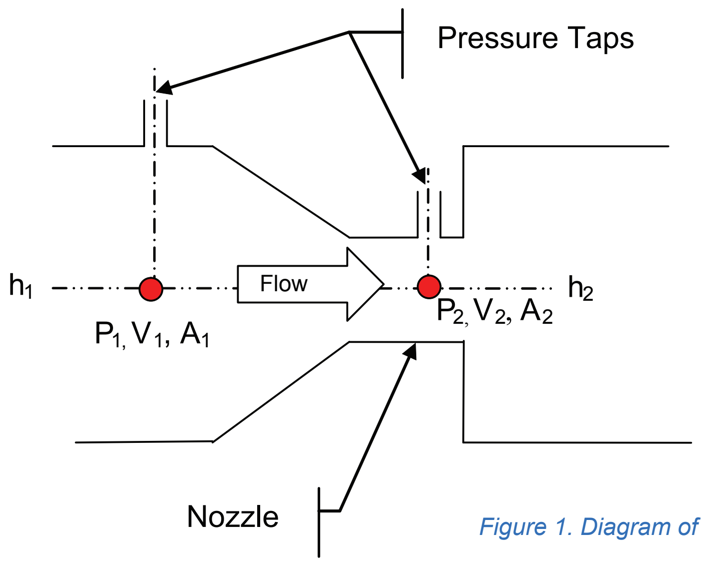


Figure 1. Diagram of a Typical Flow Meter.

It is clear from the diagram that the flow approaches and passes through a constriction, e.g., a nozzle or orifice. Pressure taps in the walls of the pipe are used to measure the pressure differential of fluid at two points (i.e., the red dots) in the flow path. The first point is upstream of the nozzle. Here,the cross-sectional area is larger, the flow is slower and the pressure is higher than at the second point, the nozzle region.

With the cross-sectional area of the upstream and nozzle regions known, along with the pressure differential, it is possible to calculate a theoretical flow rate through the pipe using conservation of mass and momentum. But, these theoretical flow rates do not account for momentum losses in the pipe, mostly in the nozzle region, due to such factors as friction and turbulence. This is where the discharge coefficient comes into play.

The discharge coefficient can be thought of as a correction factor for "real" flow meter devices. It is typically determined experimentally for different flow meters. Correlation equations can be used to calculate the real "corrected" flow rate through a given meter.

With this brief overview, it is possible to develop the basic equations used in flow meter calculations. We begin by considering the Bernoulli equation for a steady state, incompressible flow:

$$P_1+\tfrac{1}{2}\rho V_1^2+\rho g h_1=P_2+\tfrac{1}{2}\rho V_2^2+\rho g h_2 \quad (1)$$

Where:

P = Absolute pressure

V = Velocity

ρ = Density of the fluid

g = Gravitational constant

h = Height of each point in the flow path. The height difference is considered the “head pressure” in the system. For most typical flow meters this is considered negligible.

This equation can be combined with the equation of continuity:

$$\dot{m}_1=\dot{m}_2 \qquad (2)$$

Where:

$$\dot{m}=\rho VA=\rho G \qquad (3)$$

And:

$\dot{m}$ = Mass flow rate

G = Volumetric flow rate

A = Cross-sectional area at each measurement point.

The result is a general expression for the volumetric flow rate:

$$G=\left[\frac{1}{\left(\frac{1}{A_2^2}-\frac{1}{A_1^2}\right)}\frac{2}{\rho}\left[\rho g(h_1-h_2)+(P_1-P_2)\right]\right]^{1/2} \quad (4)$$

If the expression is re-written in terms of diameters, and with the head pressure considered negligible, the following simplified expression results:

$$G=\left[\frac{\pi^2}{8\rho}\left(\frac{D^4d^4}{D^4-d^4}\right)(P_1-P_2)\right]^{1/2} \qquad (5)$$

Where:

D = Larger upstream diameter of the pipe approaching the nozzle or orifice

d = Smaller nozzle or orifice diameter

To account for the frictional and turbulence related losses in the nozzle orifice the discharge coefficient is introduced.

$$G=C_d\left[\frac{\pi^2}{8\rho}\left(\frac{D^4d^4}{D^4-d^4}\right)(P_1-P_2)\right]^{1/2} \quad (6)$$

Where:

C_d = Discharge coefficient

The value of the discharge coefficient is usually determined experimentally for different types of flow meters. It is a function of the Reynolds number or the ratio of the inlet and exit cross-sectional areas. Using these discharge coefficient values along with the geometry of the flow nozzle, and the measured pressure differential it is possible to calculate the flow rate.

Some common examples of flow measurement devices with discharge coefficient correlations are shown here. The first of these is the Venturi nozzle as shown in Figure 2 [2].

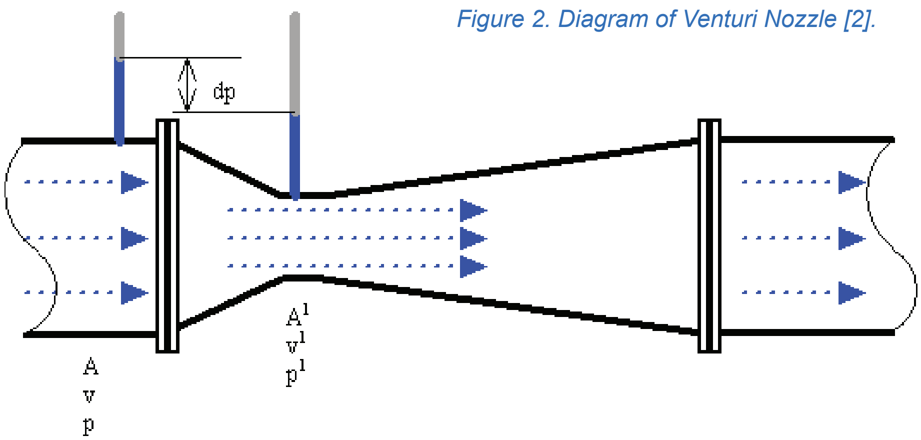


Figure 2. Diagram of Venturi Nozzle [2].

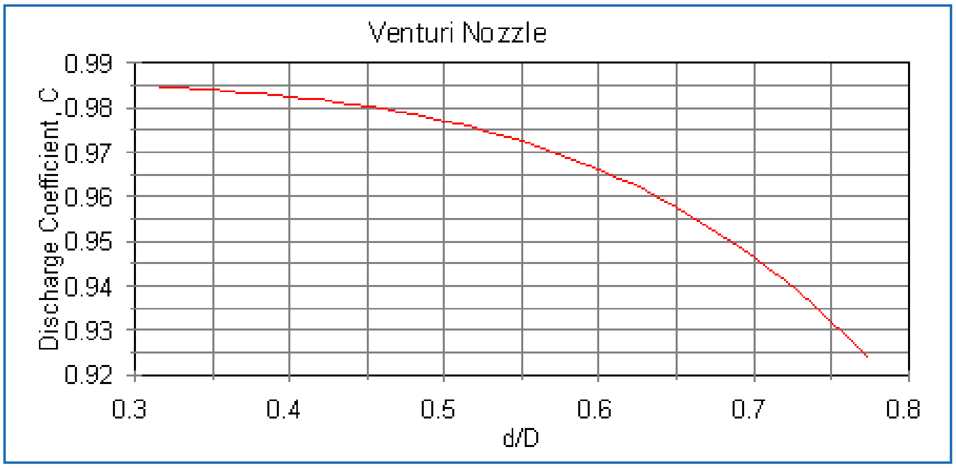


Figure 3. Plot of Discharge Coefficient as a Function of the Nozzle Diameter Ratio (d/D) [3].

The Venturi nozzle discharge coefficient equation is [3]:

$$C_d = 0.9858 - 0.196\,(d/D)^{4.5} \qquad (7)$$

Where:
The discharge coefficient equation is valid for: $0.316 \le d/D \le 0.775$, $1.5\text{x}10^5 \le ReD \le 2\text{x}10^6$, 6.5 cm ≤ D ≤ 50 cm, d ≥ 5 cm, and $k/D \le 3.8\text{x}10^{-4}$ generally.

A plot of the discharge coefficients as function of nozzle diameter ratio is shown in Figure 3.

The Venturi nozzle is excellent for low pressure head applications, as the recovery pressure for the fluid exiting the nozzle is minimal. This is due to the diverging nozzle design, which allows the fluid exiting the nozzle to return to the pre-nozzle pressure state with no vena contracta and minimal turbulence. The accuracy of the Venturi nozzle is typically 1% of full range [2].

Also popular is the ISA 1932 nozzle, which is shown in Figure 4.

The ISA 1932 nozzle discharge coefficient equation is [3]:

$$C_d = 0.99 - 0.2262(d/D)^{4.1} - [0.00175(d/D)^2 - 0.0033(d/D)^{4.15}]\,[106/Re_D]1^{15} \qquad (8)$$

Where:
The discharge coefficient equation is valid for: 5 cm ≤ D ≤ 50 cm and $0.3 \le d/D < 0.44$ having $7\text{x}10^4 \le ReD \le 10^7$ and $0.44 \le d/D \le 0.8$ having $2\text{x}10^4 \le ReD \le 107$ and $k/D \le 3.8\text{x}10^{-4}$ generally for all d/D.

Figure 4. Diagram of an ISA 1932 Nozzle [2].

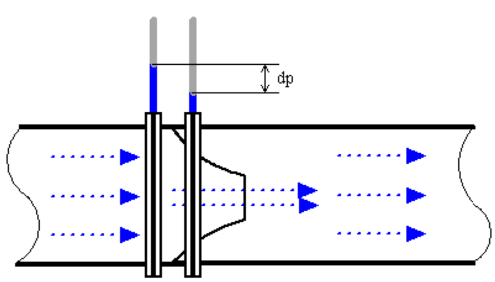
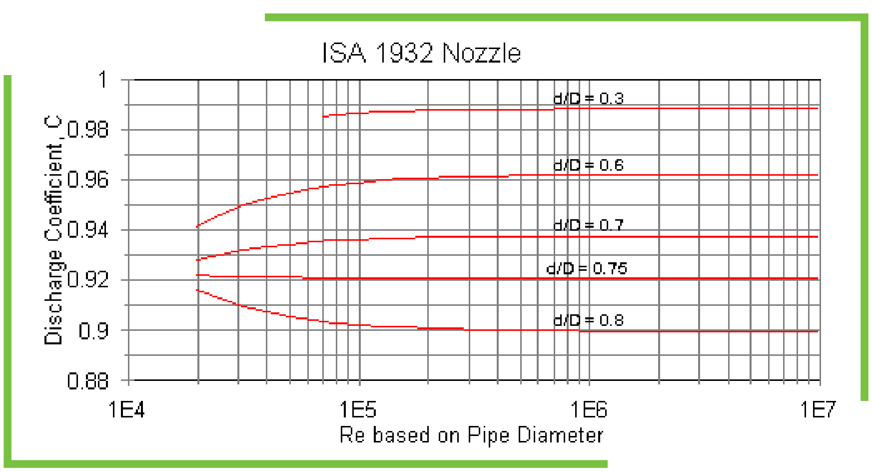


Figure 5. Plot of Discharge Coefficient as a Function of Re Number [3].



The ISA flow nozzle is another variant of the Venturi nozzle, but without the diverging section which minimizes the pressure head recovery loss. The accuracy of this nozzle is typically in the 1-2% range [2].

In conclusion, flow meters can predict the flow rate of fluids through pipes and tubes of various diameters by using the unique discharge coefficients that are determined experimentally for each flow meter type. The discharge coefficient can be thought of as a measure of how efficiently the flow meter is operating. Under ideal conditions, there will be no losses due to friction, turbulence or other fluid effects, but this is not true of real devices and systems. The discharge coefficient gives us a way to characterize these “other” effects. When used with the basic fluid mechanics equations, they can be used to solve a variety of fluid measurement problems.

References:

1. Discharge Coefficient, McGraw-Hill Dictionary of Scientific and Technical Terms. McGraw-Hill Companies, Inc., 2003.

2. Orifice, Nozzle and Venturi Flow Rate Meters, The Engineering ToolBox 2005, www.engineeringtoolbox.com/flow-meters-d_493.html, January 2008.

3. Nozzle Flowmeter Calculation for Liquids. ISO-5167 equation, LMNO Engineering, Research, and Software, Ltd., www.lmnoeng.com/nozzles.htm, January 2008.